Perturbation Finite Element Method for the Analysis of Earthing Systems with Vertical Rods

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***Abstract*—This paper deals with the electrokinetic modeling of earthing systems by means of a sub-domain perturbation finite element technique. An axisymmetric problem is solved for each single grounding rod. Its solution must then be corrected by taking into account the influence of the other rods. The electric scalar potential is transferred from one problem to the other through projections between meshes. An inherently 3D problem can thus be solved as a succession of 2D sub-problems, what significantly speeds up the solution and enables to tackle complicated grounding systems. The method is validated by means of both analytical formulas and 3D computations.**

***Index Terms*—Perturbation method, finite element methods, electrokinetics, earthing systems.**

I. Introduction

Earthing systems generally comprise several vertical rods in parallel in order to reduce the grounding resistance and enhance the safety of the low voltage equipment and personnel from the dangerous ground potential due to dissipation of fault currents or lightning discharge into the ground [1], [2]. Several analytical formulas are since long available in the literature for classical configurations [3], [4], [5]. The analysis of more complicated configurations must be done numerically and most likely in 3D. The finite element (FE) method is well suited for tackling this kind of problems. However, it may become extremely expensive due to the required dense discretization in the vicinity of the rods [6].

The perturbation FE approach allows to overcome this drawback. It has already shown to be clearly advantageous in repetitive analysis, like in nondestructive testing and moving systems applications [7], [8]. This technique takes advantage of previous computations instead of solving a completely new FE problem for any variation of geometrical or physical characteristics. Further, different problem-adapted meshes are allowed and computational efficiency is clear due to the reduced size or each sub-problem.

A perturbation FE method is herein developed for accurately calculating the resistance in earthing systems consisting of grounding rods. Each rod is defined in an axisymmetric domain and mesh. An electrokinetic FE formulation is adopted.

An axisymmetric problem is solved for each single grounding rod. Its solution must then be corrected and adapted to account for the effect of all the other rods. The electric scalar potential is transferred from one problem to the other though projections between their meshes. The successive solution of 2D axisymmetric sub-problems allows thus to solve a typically 3D problem. The method is validated by means of analytical formulas and 3D FE computations.

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II. Electrokinetic modeling of earthing systems

A. Canonical problem in a strong form

An electrokinetic problem p is defined in a domain $\Omega_p = \Omega_{c,p} \cup \Omega_{c,p}^C$ with conducting part $\Omega_{c,p}$, non-conducting part $\Omega_{c,p}^C$ and boundary $\Gamma_p = \Gamma_{e,p} \cup \Gamma_{j,p}$ (possibly at infinity), see Fig. 1. Subscript p refers to the associated problem p.

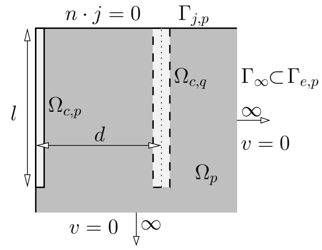

Fig. 1. Axisymmetric problem with BCs, reference domain $\Omega_{c,p}$ and perturbation domain $\Omega_{c,q}$ at distance d

The equations, material relations and boundary conditions (BCs) characterizing the electrokinetic problem p in Ω_p are:

$$\operatorname{curl} \boldsymbol{e}_p = 0\,, \quad \operatorname{div} \boldsymbol{j}_p = 0\,, \quad \boldsymbol{j}_p = \sigma_p\, \boldsymbol{e}_p\,, \qquad \text{(1 a-c)}$$

$$\boldsymbol{n} \times \boldsymbol{e}|_{\Gamma_{e,p}} = 0\,, \quad \boldsymbol{n} \cdot \boldsymbol{j}|_{\Gamma_{j,p}} = 0\,, \qquad \text{(1 d e)}$$

where $\boldsymbol{e}_p$ is the electric field, $\boldsymbol{j}_p$ is the electric current density, σ_p is the electric conductivity and $\boldsymbol{n}$ is the unit normal exterior to Ω_p. According to (1 a), the electric field can be expressed in terms of an electric scalar potential v, i.e. $\boldsymbol{e}_p = -\operatorname{grad} v_p$. The BC (1 d) defines a constant scalar potential on each non-connected part of $\Gamma_{e,p}$. It is applied on the boundary $\Gamma_{c,p}$ of each perfect conductor $\Omega_{c,p}$ and on the infinity boundary Γ_∞ of Ω_p.

At the discrete level, independent meshes are used for all problems p. Infinity is taken into account by means of a geometrical transformation [9].

B. Perturbation problems

A modification of an initial problem $p = 1$ due to a change of conductivity and/or an addition of sources in some sub-regions leads to the perturbation of the field quantity. Both large and small perturbations can be accounted for (e.g. change of properties of materials, adding new materials [10])). In earthing systems, the perturbing regions will then be additional grounding rods that influence the initial electric field distribution.

The perturbation FE method consists thus in determining the solution of P successive sub-problems $p = 1, \ldots, P$, the addition of which being the solution of the complete problem. The complete solution is then:

$$v = \sum_{p=1}^{P} v_p\,, \quad \boldsymbol{e} = \sum_{p=1}^{P} \boldsymbol{e}_p\,, \quad \boldsymbol{j} = \sum_{p=1}^{P} \boldsymbol{j}_p\,. \qquad \text{(2 a–c)}$$

As each sub-problem is generally perturbed by all the others, each solution v_p has to be calculated as a series of corrections, i.e.

$$v_p = v_{p,1} + v_{p,2} + \cdots\,. \qquad (3)$$

The calculation of the corrections $v_{p,i}$ in a problem p, i is kept on till convergence up to a desired accuracy. Each correction $v_{p,i}$ must account for the influence of all the previous corrections $v_{q,j}$ of the other sub-problems, with $q = 1, \cdots, p-1, j = i$ and $q = p+1, \cdots, P, j = i-1$. Further, initial solutions $v_{p,0}$ are set to zero.

In our case, the added region $\Omega_{c,p}$ is a perfect conductor. This allows to determine the source of each perturbation problem p, i by taking into account that total electric field must be zero in $\Omega_{c,p}$, $\boldsymbol{e}|_{\Omega_{c,p}} = 0$. The source of each problem p, i in $\Omega_{c,p}$, which can be also written in terms of the electric scalar potential $v_{p,i}$, is given thus by

$$\boldsymbol{e}_{p,i} = -\sum_{\substack{q=1\\ q\neq p}}^{P} \boldsymbol{e}_{q,j}\,, \quad v_{p,i} = -\sum_{\substack{q=1\\ q\neq p}}^{P} v_{q,j}\,, \ \text{in } \Omega_{c,p}\,, \qquad \text{(4 a b)}$$

where j is the last iteration index for which the associated solution is known. It is worth mentioning that (4 b) can be limited to $\Gamma_{c,p}$, defining a Dirichlet BC.

The method is also valid when dealing with the inclusion of conductors with a finite conductivity. An additional volume source must then be added to (1 c) [10].

For the sake of simplicity, we consider herein only vertical grounding rods. In this particular case, each $\boldsymbol{j}_{p,i}$ verifies automatically BC (1 e) which also holds for the complete $\boldsymbol{j}$ (principle of superposition). The method is nevertheless valid for arbitrary orientations. For orientations other than vertical, BC (1 e) should be corrected, e.g. by applying image theory.

III. Weak Finite Element Formulation

A. *Canonical problem in a weak form*

The electric scalar potential formulation of the electrokinetic problem p (1) is given by

$$(\sigma_p \operatorname{grad} v_p, \operatorname{grad} v')_{\Omega_p} + \langle \boldsymbol{n}\cdot\boldsymbol{j}_p, v'\rangle_{\Gamma_p} = 0\,, \quad \forall\, v' \in F_v(\Omega_p)\,, \qquad (5)$$

where $(\cdot,\cdot)_\Omega$ and $\langle\cdot,\cdot\rangle_\Gamma$ denote, respectively, a volume integral in Ω and a surface integral on Γ of the scalar product of their arguments; $F_v(\Omega_p)$ is the function space defined on Ω_p and containing the basis functions for v_p as well as for the test function v' [11]. At the discrete level, $F_v(\Omega_p)$ is approximated with nodal FEs.

A global basis function is associated to each non-connected portion of $\Gamma_{e,p}$. It equals one on this portion and varies continuously in Ω_p up to zero on the other portions [11]. At the discrete level, such a function can be defined as the sum of the nodal FE basis functions of the nodes of the boundary portion. Such a function, when applied as test function v' in (5), allows to determine the current flowing from the associated boundary. Resistances are then straightforwardly calculated from the values of voltages and currents.

Formulation (5) is valid for any correction $v_{p,i}$ of (3) involved in the iterative process. The associated BC (4 b) on $\Gamma_{c,p}$ has to be strongly defined in $F_v(\Omega_p)$. Each solution $v_{p,i}$ leads to a correction of the current and consequently of the resistance to ground of $\Omega_{c,p}$.

B. *Projection of sources*

Each grounding rod is modeled by an axisymmetric and independent mesh. Consequently, each source scalar potential $v_{q,j}$ in BC (4 b) initially interpolated in the mesh of problem q has to be transferred to the mesh of problem p. This is done via a projection method [12].

Let $v_q(\boldsymbol{x}_p)$ denote thus the electric scalar potential calculated in Ω_q and evaluated at a certain position $\boldsymbol{x}_p$ in Ω_p. Given the perfectly conducting nature of the perturbing regions, the projection $v_{q_{proj}}$ of v_q from its original mesh to that of $\Omega_{c,p}$ is restricted to its boundary $\Gamma_{c,p}$. It reads:

$$\langle \operatorname{grad} v_{q_{proj}}, \operatorname{grad} v'\rangle_{\Gamma_{c,p}} = \langle \operatorname{grad} v_q(\boldsymbol{x}_p), \operatorname{grad} v'\rangle_{\Gamma_{c,p}}\,, \quad \forall\, v' \in F_v(\Gamma_{c,p})\,, \qquad (6)$$

where the function space $F_v(\Gamma_{c,p})$ contains v_q and its associated basis functions v'. For the sake of simplicity, $v_{q_{proj}}$ will be referred to as v_q. At the discrete level, v_q is discretized with nodal FEs and it is linked to a gauge condition fixing a nodal value in $\Gamma_{c,p}$.

Because each mesh is defined in its own coordinate system, a geometrical transformation is required in the projection process. A point $\boldsymbol{x}_p$ in Ω_p must be transformed to a point $\boldsymbol{x}_q$ in Ω_q via a transformation Ψ_{pq}. The source potential in (6) is then given by

$$v_q(\boldsymbol{x}_p) = v_q(\Psi_{pq}(\boldsymbol{x}_p)) = v_q(\boldsymbol{x}_q)\,, \ \boldsymbol{x}_p \in \Omega_p\,, \boldsymbol{x}_q \in \Omega_q\,. \qquad (7)$$

For a set of problems with vertical rods, the transformation is just $\Psi_{pq}(\boldsymbol{x}_p) = \boldsymbol{x}_p + d_{pq}$, d_{pq} being the distance between the rods p and q. The axisymmetric nature of each sub-problem implies $\Psi_{pq}(\boldsymbol{x}_p) = d_{pq}$.

In case of a non-perfectly conducting perturbing region, the projection should be extended to the whole domain $\Omega_{c,p}$. We choose to directly project $\operatorname{grad} v_q$ to guarantee a better numerical behaviour of the ensuing equations where the involved quantities are also gradients.

Note that when dealing with identical rods (same dimensions), the working mesh is in fact the same for all of them.

IV. Application Example

The perturbation FE method is validated by comparing its results to those obtained by either analytical formulas or 3D FE computations.

Even though considering a problem with just one grounding rod has no interest in a perturbation approach, the error committed when applying approximated analytical formulas or the 3D FE method must be somehow quantify. Therefore, we start our validation by considering a single grounding rod (radius $r = 1.25\,\mathrm{cm}$, length l varying between 1–10 m) driven in a homogeneous soil with resistivity $\rho = 100\,\Omega\,\mathrm{m}$ and subjected to a given voltage. This problem is solved with an axisymmetric FE model, which is in fact the initial solution in the perturbation FE technique.

The resistance to ground for a single grounding rod may be calculated from

$$R = \frac{\rho}{2\pi r_{eq}} \tag{8}$$

with ρ the soil resistivity in $\Omega\,\mathrm{m}$. Different expressions can be found in the literature for r_{eq} as well. We have used the Rudenberg formula $r_{eq} = \frac{l}{\ln\frac{2l}{r}}$ [4], the Dwight-Sunde formula $r_{eq} = \frac{l}{\ln\frac{4l}{r}-1}$ [5] and the Liew-Darveniza formula $r_{eq} = \frac{l}{\ln\frac{r+l}{r}}$ [13].

For the sake of a fair comparison between the perturbation (axisymmetric) and the 3D FE results, the perturbation FE computations (when a 3D solution is shown) have been performed with both a fine and a coarse mesh (see detail of the discretization in the vicinity of the tip of the rod in Fig. 2). Indeed, the level of refinement of the mesh on the left is prohibitive in a 3D computation. The 3D mesh used in our calculations is as coarse as the coarse axisymmetric mesh depicted in Fig. 2.

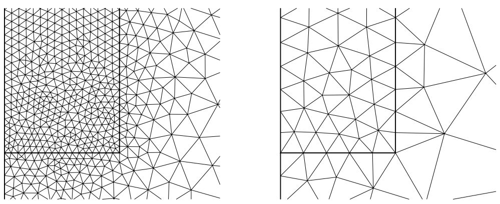

Fig. 2. Detail of the axisymmetric fine mesh (left) and coarse mesh (right) in the vicinity of the tip of the rod

The values of the resistance to ground of a single rod as a function of its length obtained with these analytical formula, an axisymmetric FE model and a 3D FE model are shown in Fig. 3 (up). The 3D FE mesh is roughly 4 times coarser than the axisymmetric mesh around the tip of the grounding rod. Taking the axisymmetric model as reference, we can compute the relative error of the resistance given by these analytical expressions and the 3D model. This error is also represented in Fig. 3 (down). For all considered analytical formulas, the error increases with the length of the grounding rod. The Liew-Darveniza expression seems to be more accurate.

We consider now different configurations of these vertical copper rods subjected to a common voltage. The distance between two consecutive rods is taken as twice their length, i.e. $d = 2\,l$ (Fig. 4).

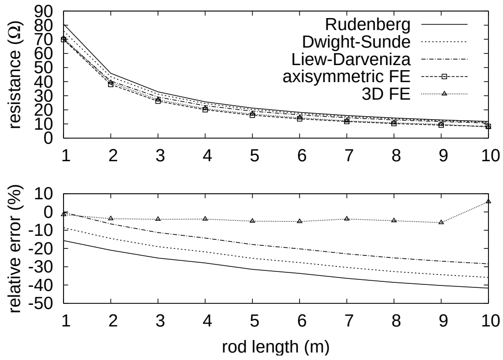

Fig. 3. Earth resistance versus rod length obtained by analytical formulas, an axisymmetric FE model and a coarse 3D model (up). Relative error with regard to the 2D axisymmetric FE solution (down)

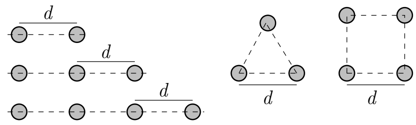

Fig. 4. Grounding rod configurations: in line (left), equilateral triangle (center), quadrangle (right). Distance between consecutive rods $d = 2\,l$

The proposed perturbation method allows us to avoid a cumbersome and often unfeasible 3D mesh operation (e.g. in case of complicated grounding systems) by using as support only axisymmetrical meshes. This way, a high accuracy also is ensured.

A. Aligned rods

Given that the used analytical formulas are not exact (see Fig. 3) and that the level of refinement of the 3D mesh is limited, we take as a reference the result obtained by a perturbation FE model with a sufficiently fine mesh. The relative error shown is thus computed with respect to this reference.

The first test case consists of two aligned vertical grounding rods. The electric scalar potential distribution achieved with both the 3D model (left) and the perturbation model (right) is depicted in Fig. 5. Given that the perturbation FE approach uses exactly the same axisymmetric mesh for both rods, the electric scalar potential map for the second rod coincides with the one already shown.

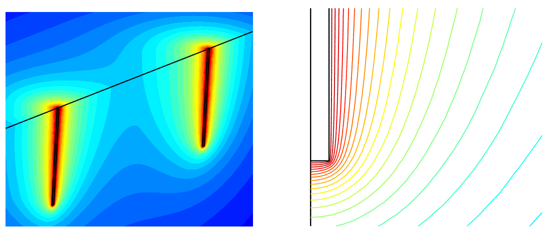

Fig. 5. Detail of the final electric potential distribution around grounding rods in a two-electrode configuration: 3D FE result (left), perturbation FE result in axisymmetric model (right)

In general, the resistance to ground of N aligned grounding rods can be approximated by the following expression [14]:

$$R_N \approx \frac{1}{N}\left[\frac{\rho}{2\pi l}\left(\ln\frac{2l}{r}-1\right)+\frac{\rho}{\pi d}\left(\frac{1}{2}+\cdots+\frac{1}{N}\right)\right]. \quad (9)$$

The values of the resistance as a function of the rod length given by this analytical formula are compared with those obtained by the perturbation FE method (fine and coarse mesh) and the 3D FE method in Fig. 6. The relative error of all these results with regard to the so-considered reference perturbation model is also shown. As expected, the errors committed with the 3D model and the coarse perturbation model are very similar. These two meshes are clearly not fine enough.

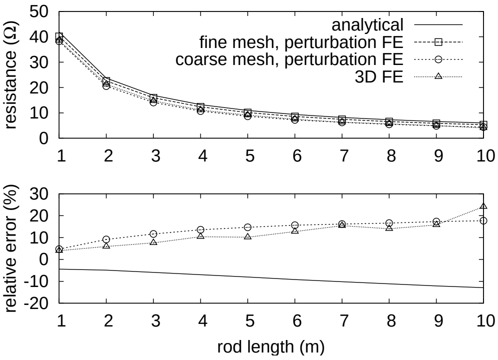

Fig. 6. Earth resistance versus rod length obtained by an analytical formula, a perturbation FE method (fine and coarse mesh) and a 3D FE method for a configuration of two aligned grounding rods (up). Relative error with regard to the fine perturbation FE solution (down)

Very similar results for the resistance and the relative error are shown in Fig. 7 and Fig. 8 where three and four aligned rods are considered as test case. Again, the analytical formula (9) has been used. The 3D FE model gives again results very close to those of the coarse perturbation model that are not accurate at all when the length of the rod increases.

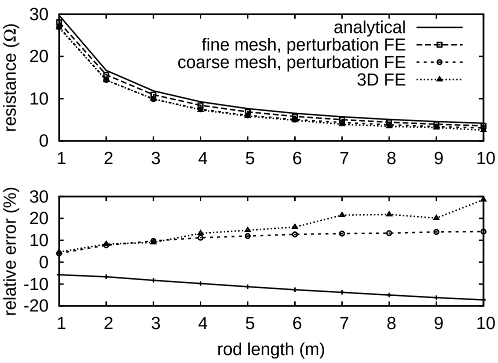

Fig. 7. Earth resistance versus rod length obtained by an analytical formula, a perturbation FE method (fine and coarse mesh) and a 3D FE method for a configuration of three aligned grounding rods. Relative error with regard to the fine perturbation FE solution (down)

B. Triangular and quadrangular rod configurations

The perturbation FE method has also been successfully applied to three rods disposed in an equilateral triangular configuration and to four rods in a quadrangular configuration.

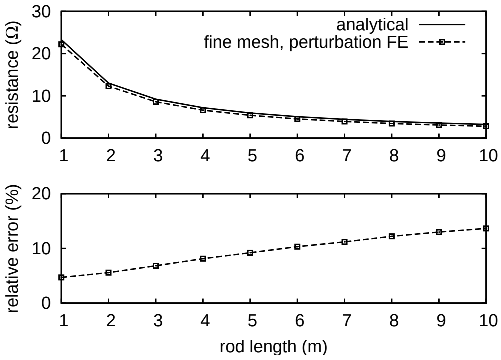

Fig. 8. Earth resistance versus rod length obtained by an analytical formula and a perturbation FE method for a configuration of four aligned grounding rods (up). Relative error with regard to the perturbation FE solution (down)

For grounding rods placed at the corners of an equilateral triangle or at the corners of a square, we have, respectively [14]:

$$R_{3t} = R\,\frac{d+2\,r_{eq}}{3\,d}, \qquad R_{4q} = R\,\frac{d+2.7071\,r_{eq}}{4\,d}, \quad (10)$$

with R the resistance for a single rod (8), d the distance between the rods and r_{eq} the radius of the hemisphere equivalent to a cylindrical vertical rod.

The resistance values together with the relative errors in terms of the length of the rods for the triangular and the quadrangular configurations are given in Figs. 9 and 10. In both configurations, the analytical formulas underestimate the value of the earth resistance. Further, the Liew-Darveniza formula provides the more precise results as in the one rod validation case.

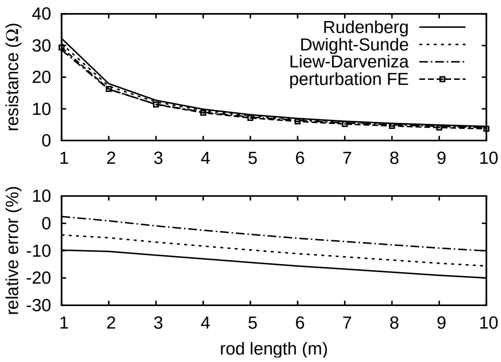

Fig. 9. Earth resistance versus rod length obtained by analytical formulas and a perturbation FE method for three grounding rods in equilateral triangular configuration(up). Relative error with regard to the perturbation FE solution (down)

C. Computation cost

In order to highlight the efficiency of the proposed perturbation method, we analyze the computational data of the case of two identical one meter length aligned rods separated by a distance of two meters.

The system of algebraic equations is solved by means of the iterative solver GMRES [15] with ILU-preconditioning on a 2.26 GHz Intel Pentium M Processor.

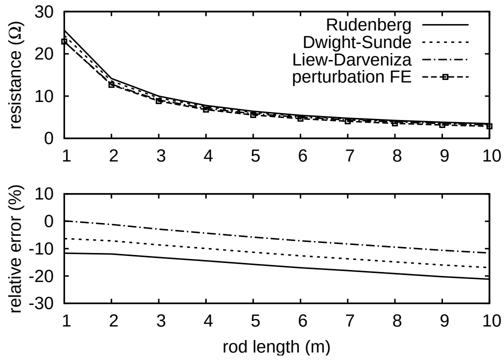

Fig. 10. Earth resistance versus rod length obtained by analytical formulas and a perturbation FE method for four grounding rods in a quadrangular configuration(up). Relative error with regard to the perturbation FE solution (down)

TABLE I
COMPUTATION TIME FOR THE CONVENTIONAL FE APPROACH AND THE PERTURBATION FE METHOD (COARSE AND FINE MESHES)

	3D FE		Pert. FE (coarse)		Pert. FE (fine)	
l (m)	N	t (s)	N_{per}	t (s)	N_{per}	t (s)
1	52206	77.13	1385	0.62	3011	2.04
2	51565	91.08	1515	0.70	3471	2.46
3	54005	100.81	1624	0.76	3941	3.13
4	53932	99.21	1718	0.83	4030	3.32
5	52229	103.72	1750	0.84	4167	3.44
6	55312	122.03	1815	0.89	4364	3.70
7	55586	113.68	1824	0.89	4512	3.75
8	55788	123.15	1876	0.94	4457	3.82
9	56280	131.56	1859	0.92	4586	3.89
10	53984	138.98	1965	1.00	4693	4.21

The 3D FE conventional approach employs as support a semi-hemispherical mesh that yields N scalar unknowns. When applying the perturbation scheme, a single axisymmetrical mesh (coarse or fine) is used for accounting for the two rods. Each sub-problem and projection yield N_{per} scalar unknowns. Convergence is achieved after 3 iterations (for the configuration at hand in this section), where each iteration required 2 perturbation corrections for a desired relative accuracy of 1%.

With the coarse perturbation model and for all considered dimensions, the number of unknowns is reduced by a factor between 27 and 38; the computation time is also improved by a factor between 120 and 143. A significant speed-up is thus achieved with the same accuracy.

In order to increase this accuracy, a finer mesh can be envisaged with the perturbation FE method. Even then, the method proves to be extremely efficient. Check Table I for more details. Ensuring the same level of precision with a 3D model would be roughly four times more expensive than with the 3D model at hand.

Moreover, nothing has been mentioned yet concerning the time required for meshing. Indeed, this task becomes expensive in 3D cases, the meshing time being non-negligible with regard to the computation time.

These results clearly illustrate the efficiency of the method in terms of memory requirements and computational time. The high-performance of the perturbation FE technique makes it extremely powerful and attractive in parameterized studies.

V. CONCLUSIONS

The electrokinetic analysis of earthing systems by means of a sub-domain perturbation finite element technique has been elaborated. The method allows to uncouple different FE regions which significantly simplifies the meshing process and reduces the computational cost. In particular, each grounding rod is considered independently with an associated axisymmetric domain and mesh.

The solution of each sub-problem is successively corrected to account for the influence of the additional grounding rods. The electric scalar potential is transferred from one problem to the other through projections between the meshes.

An inherently 3D problem can thus be solved as a succession of 2D sub-problems, what significantly speeds up the solution and enables to tackle the computation of the earth resistance in complicated grounding systems. The method has been validated by means of both analytical formulas and 3D computations.

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